

Performance Analysis of Channel Estimation and Adaptive Equalization in Slow Fading Channel

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ABSTRACT

In our project, we first build up a wireless communication simulator including Gray coding, modulation, different channel models (AWGN, flat fading and frequency selective fading channels), channel estimation, adaptive equalization, and demodulation. Next, we test the effect of different channel models to the data and image in receiver with constellation and BER (bit error rate) plots under QPSK modulation. For Image data source, we also compare the received image quality to original image in different channels. At last, we give detail results and analyses of the performance improvement with channel estimation and adaptive equalization in slow Rayleigh fading channel. For frequency selective fading channel, we use linear equalization with both LMS (least mean squares) and RLS (Recursive Least Squares) algorithms to compare the different improvements. We will see that in AWGN channel, the image is degraded by random noise; in flat fading channel, the image is degraded by random noise and block noise; in frequency selective fading channel, the image is degraded by random noise, block noise, and ISI.

Keywords: *Slow fading, flat fading, frequency selective fading, channel estimation, LMS, RLS*

I. INTRODUCTION

Mobile communications and wireless network have experienced massive growth and commercial success in the recent years. However, the radio channels in mobile radio systems are usually not amiable as the wired one. Unlike wired channels that are stationary and predictable, wireless channels are extremely random and time-variant. It is well known that the wireless multi-path channel causes an arbitrary time dispersion, attenuation, and phase shift, know as fading, in the received signal. Fading is caused by interference between two or more versions of the transmitted signal which arrive at the receiver at slightly different times.

There are many diversity techniques to address fading issue, such as OFDM, MIMO, RAKE receiver and etc. However, it may be still necessary to remove the amplitude and phase shift caused by the channel if you want to apply linear modulation schemes, such as the ones used in WiMAX. The function of channel estimation is to form an estimate of the amplitude and phase shift caused by the wireless channel from the available pilot information. Channel estimation methods may be divided into two classes: pilot-based estimation and blind estimation. In our project, we will focus on pilot-based channel estimation

with training data. The equalization removes the effect of the wireless channel and allows subsequent symbol demodulation. An adaptive equalizer is a time-varying filter which must constantly be retuned. A number of different algorithms can be employed for these modules. In our project, we use LMS (least mean squares) and RLS (Recursive Least Squares).

Digital communication systems operating on time varying dispersive channels often employ a signaling format in which customer data are organized in blocks preceded by a known training sequence. The training sequence at the beginning of each block is used to estimate channel or train an adaptive equalizer. Depending on the rate at which the channel changes with time, there may or may not be a need to further track the channel variations during the customer data sequence.

Fig.1 shows the flow chart of our Matlab simulation which is used in this project.

II. SYSTEM MODEL AND MAIN TECHNIQUES

A. To build up our Model

In our project, our target is to build up a slow fading channel for both flat fading and frequency selective fading, so we choose two different environments to simulate them, as described below:

For *slow fading*, $T_s \ll T_c$, and for *flat fading*, $T_s \gg \sigma_\tau$ (equation 5.42 in Rappaport textbook), so we get $\sigma_\tau \ll T_s \ll T_c$ for a slow flat fading channel. For frequency selective fading, $T_s < \sigma_\tau$, so we get $T_s < \sigma_\tau < T_c$ for slow frequency selective fading channel.

We choose a model simulated to GSM, where the carrier frequency is 1.8GHz, and bandwidth of each channel is 200KHz. Suppose we use Nyquist pulse to transmit, we get $T_s = 5$ micro second, where T_s is symbol period. We simulate two scenarios: in the first scenario, we simulate an urban environment, where RMS delay spread is 10-25us (Table 5.1 in Rappaport textbook) and we choose delay spread as 10 us. Now suppose the user is walking in a velocity of 5km/hr. so $T_c = 9 / (16 * \pi * fm)$ (equation 5.40.b in Rappaport textbook) $= 21.5ms \gg 10us > 5us$, it is a slow frequency selective fading channel. In the second scenario, we simulate a suburb environment, where RMS delay spread is 200-310ns (Table 5.1 in Rappaport textbook) and we choose delay spread as 300ns.

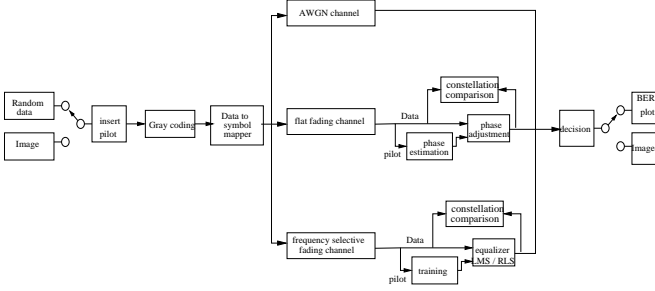


Figure 1. The flow chart of Matlab simulation

Now suppose the user is on a training which has a velocity of 20-120km/hr. (actually we may set the velocity from 0-1000km/hr to simulate a slow fading channel, but it is out of the practical scenario and may out of Matlab processing capability, so we let the velocity in the range from 20km/hr to 120km/hr) We will test two different velocity in this scenario for 20km/hr and 120km/hr. For 20km/hr, we get $T_c = 5.4\text{ms} \gg 5\mu\text{s} \gg 300\text{ns}$, so it is a slow flat fading channel. For 120km/hr, we get $T_c = 900\mu\text{s} \gg 5\mu\text{s} \gg 300\text{ns}$, so it is also a slow flat fading channel.

In both above two scenario, we suppose there are no dominant stationary (non-fading) signal component present at receiver side, such as a line-of-sight propagation path, and the fading follow a Rayleigh distribution, so both of them are slow Rayleigh fading channel.

B. To produce data and set parameter

Since the baseband complex envelope expression can be used to represent bandpass waveforms, the channel response, demodulated signal, and adaptive algorithms are usually simulated and implemented at baseband. So, our simulations are built up by baseband simulation.

Our simulation supports two kinds of data source, either randomly produced data or an image file. While random data is ideal to test the BER performance and channel effect to signal constellation, image files give us an intuitive impression and comparison for different channels. In our model, we use phase shift keying (PSK) modulation to modulate the data source, and user may choose arbitrary M-ary PSK to modulate the signal. In our simulation, we test the QPSK modulation. User may also have an option to select using gray coding or not in the simulation.

Since we choose PSK modulation, we need to estimate channel phase during each coherence time. In our model, the pilot data length is 8% of the total data length and is inserted into head of source data in each coherence time. It is used to estimate the random phase shift of the fading channel and train the decision to adjust the received signal with phase recover. The received signal constellations of both with and without phase recover are dynamically showed in the simulation. User may choose to plot constellation or not in the program, and if they choose to plot constellation, they may also set the SNR for the constellation.

C. To produce different channels

In our simulation, we simulate three different channels:

AWGN channel, slow flat Rayleigh fading channel, and slow frequency selective Rayleigh fading channel. While AWGN channel is very straightforward by just add a white Gaussian noise into signal to meet specified SNR, we need to derive the fading channel to meet our requirement. In Matlab program, MyRayleigh.m and MyRayleighPDP.m, a simulation of Clarke and Gans fading model is produced. Here, I will derive the calculations and equations which are used in both the flat fading channel impulse response in MyRayleigh.m and frequency selective fading channel impulse response in MyRayleighPDP.m.

1) For flat fading channel

We note that in the Rappaport textbook 5.7.2, it produces complex Gaussian random variables by step 3 and step 4 in page 222. While in MyRayleigh.m, N samples of iid complex Gaussian random variable are produced by directly generating $\lfloor N * f_d / f_s \rfloor$ numbers^[1] of iid complex Gaussian random variable in frequency domain. The other steps are same as in the textbook. The proof of these N samples complex Gaussian random variable in time domain are still all iid is not provided in this document.

I would derive the calculation by below three steps:

1. To explain that, in flat fading channel with N samples, $E[\|\bar{\mathbf{h}}\|^2] = N$, where $\bar{\mathbf{h}}$ represent channel impulse response with N samples.
2. To calculate the channel impulse response before normalization, denoted as $\bar{\mathbf{h}}_0$.
3. To calculate the channel impulse response after normalization, denoted as $\bar{\mathbf{h}}$.

The more details of these three steps are described as below.

First Step:

In a fading channel with impulse response of L bins,

$$E[\|\bar{\mathbf{h}}_L\|^2] = \sum_{i=0}^{L-1} E[|a_i|^2] \quad (1)$$

where a_i is the ith tap of impulse response in $\bar{\mathbf{h}}_L$ ^[2] (May refer to equation 5.19 and 5.24 in Rappaport Textbook). In a flat fading channel, since there is only one tap in channel impulse response \mathbf{h} , for each sample of \mathbf{h} , the expectation value of \mathbf{h} is $E[|\mathbf{h}|^2] = E[|a_0|^2]$.

To get the comparable SNR to a non-fading AWGN channel, $E[|\mathbf{h}|^2] = 1$. So, in a flat fading channel, the expectation value of $\bar{\mathbf{h}}$ with N samples is

$$E[\|\bar{\mathbf{h}}\|^2] = N * E[|\mathbf{h}|^2] = N$$

so we get:

^[1] It is easy to prove that in N points IFFT, with sampling rate f_s and Doppler shift f_d , there are only $\lfloor N * f_d / f_s \rfloor$ points of non-zero value in frequency domain.

^[2] Note: $\bar{\mathbf{h}}_L$ represent the impulse response with L bins in a frequency selective fading channel, and it should be conceptually differentiated with N samples of impulse response with only 1 tap in a flat fading channel.

$$E[\|\vec{\mathbf{h}}\|^2] = N \quad (2)$$

Second step:

Refer to Rappaport textbook 5.7.1.1, we may get RF signal spectral shape after Doppler spread by the product of amplitude of complex Gaussian random variable and root square of Doppler power spectrum. We denote $\vec{\mathbf{A}}$ as amplitude of complex Gaussian random variable in frequency domain, $\vec{\mathbf{D}}$ as root square of Doppler power spectrum (In MyRayleigh.m, they are denoted as `ampl` and `sqrtpsd` respectively), and $\vec{\mathbf{H}}_b$ as the RF signal spectral shape after Doppler spread. So, $\vec{\mathbf{H}}_b$ is actually a scaled version of $\vec{\mathbf{D}}$, where each point in $\vec{\mathbf{A}}$ is a scaler of complex Gaussian random variable. To represent $\vec{\mathbf{H}}_b$ by product of $\vec{\mathbf{D}}$ and $\vec{\mathbf{A}}$, we should use a diagonal matrix with each component in the main diagonal come from $\vec{\mathbf{A}}$. Therefore, we get:

$$\vec{\mathbf{H}}_b = \Sigma \cdot \vec{\mathbf{D}}, \text{ where } \Sigma \sim \text{CN}(0, \sigma^2) \quad (3)$$

(In MyRayleigh.m, $\Sigma \cdot \vec{\mathbf{D}}$ is calculated by `ampl .* sqrtpsd`), where Σ is a diagonal matrix and each component in the main diagonal is a complex Gaussian random variable, which is produced by `randn()` function, so variance of all components in the main diagonal are $2\sigma^2 = 2$. Thus, the expectation value of Σ is $E[\Sigma^2] = 2\sigma^2 \cdot \mathbf{I} = 2 \cdot \mathbf{I}$, where \mathbf{I} is a identity matrix. By equation (3),

$E[\|\vec{\mathbf{H}}_b\|^2] = E[\|\Sigma \cdot \vec{\mathbf{D}}\|^2] = \sigma^2 \cdot \|\vec{\mathbf{D}}\|^2 = 2 \cdot \|\vec{\mathbf{D}}\|^2$, so we get:

$$E[\|\vec{\mathbf{H}}_b\|^2] = 2 \cdot \|\vec{\mathbf{D}}\|^2 \quad (4)$$

Now we calculate $\vec{\mathbf{h}}_b$ by

$$\vec{\mathbf{h}}_b = \mathbf{F}^{-1}\{\vec{\mathbf{H}}_b\} \quad (5)$$

According to Parseval's theorem for Discrete Fourier Transform,

$$\|\vec{\mathbf{h}}_N\|^2 = \sum_{n=0}^{N-1} |h_n|^2 = \frac{1}{N} * \sum_{k=0}^{N-1} |H_k|^2 = \frac{1}{N} * \|\vec{\mathbf{H}}_N\|^2$$

and we get:

$$\|\vec{\mathbf{h}}_b\|^2 = \frac{1}{N} * \|\vec{\mathbf{H}}_b\|^2 \quad (6)$$

With equation (4),

$$E[\|\vec{\mathbf{h}}_b\|^2] = \frac{1}{N} * E[\|\vec{\mathbf{H}}_b\|^2] = \frac{2}{N} \cdot \|\vec{\mathbf{D}}\|^2$$

so we get:

$$E[\|\vec{\mathbf{h}}_b\|^2] = \frac{2}{N} \cdot \|\vec{\mathbf{D}}\|^2 \quad (7)$$

Third step:

To normalize $\vec{\mathbf{h}}_b$ to get equation (2), we set $\vec{\mathbf{h}} = \lambda \cdot \vec{\mathbf{h}}_b$. λ is calculate as below. By equation (2), we have

$$E[\|\vec{\mathbf{h}}\|^2] = E[\|\lambda \cdot \vec{\mathbf{h}}_b\|^2] = \lambda^2 \cdot E[\|\vec{\mathbf{h}}_b\|^2] = N$$

We get

$$\lambda = \sqrt{\frac{N}{E[\|\vec{\mathbf{h}}_b\|^2]}} \quad (8)$$

and

$$\vec{\mathbf{h}} = \lambda \cdot \vec{\mathbf{h}}_b = \frac{\sqrt{N} \cdot \vec{\mathbf{h}}_b}{\sqrt{E[\|\vec{\mathbf{h}}_b\|^2]}} \quad (9)$$

With equation (7), We finally get:

$$\vec{\mathbf{h}} = \frac{\sqrt{N} \cdot \vec{\mathbf{h}}_b}{\sqrt{\frac{2}{N} \cdot \|\vec{\mathbf{D}}\|^2}} \quad (10)$$

2) For frequency selective fading channel

In MyRayleighPDP.m, we produce a frequency selective fading channel with impulse response of L bins, where each tap is a flat fading channel scaled by an exponential PDP (Power Delay Profile). We may use the same method come from MyRayleigh.m to derive $\vec{\mathbf{h}}$ for each tap. However, we note that in a frequency selective fading channel, $E[|a_i|^2] \neq 1$, where a_i is the i th tap of impulse response in $\vec{\mathbf{h}}_L$, and instead, we should use the following equation

$$E[\|\vec{\mathbf{h}}_L\|^2] = \sum_{i=0}^{L-1} E[|a_i|^2] = 1$$

as mentioned above. Since the channel has an exponential PDP, the expectation value of each bin is

$$E[|a_i|^2] = \alpha \cdot e^{-\frac{iT}{\tau}} \quad (11)$$

where T is the sampling period, and τ is the time constant. So, we get

$$\sum_{i=0}^{L-1} E[|a_i|^2] = \sum_{i=0}^{L-1} \alpha \cdot e^{-\frac{iT}{\tau}} = \alpha \cdot \sum_{i=0}^{L-1} e^{-\frac{iT}{\tau}} = 1 \quad (12)$$

and

$$\alpha = \frac{1}{\sum_{i=0}^{L-1} e^{-\frac{iT}{\tau}}} \quad (13)$$

Now we may calculate a_i from h_i , which come from the result of MyRayleigh.m, that is $E[\|\vec{\mathbf{h}}\|^2] = N$, and h_i is each sample in the N sample vector $\vec{\mathbf{h}}$, So $E[|h_i|^2] = 1$. With equation (11), it is easy to get

$$a_i = \sqrt{\alpha \cdot e^{-\frac{iT}{\tau}}} \cdot h_i \quad (14)$$

D. Equalization algorithm

The performance of an estimation algorithm is determined by various factors, which include:

1. Rate of convergence – This is defined as the number of iterations required for the algorithm, in response to sta-

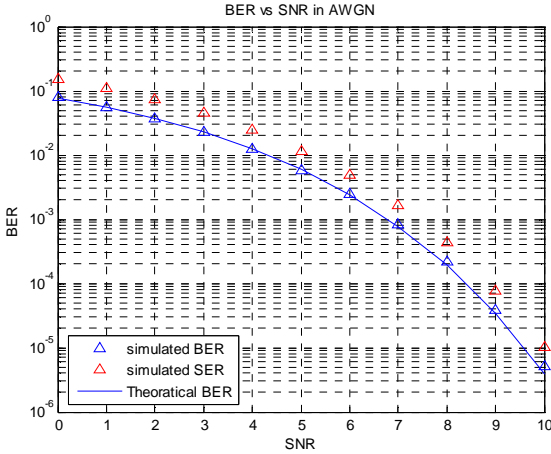


Figure 2: BER of simulation vs theoretical
 tional inputs, to converge close enough to the optimum
 solution. A fast rate of convergence allows the algorithm to
 adapt rapidly to a stationary environment of unknown
 statistics. Furthermore, it enables the algorithm to track
 statistical variations when operating in a non-stationary
 environment

2. Maladjustment – For an algorithm of interest, this parameter provides a quantitative measure of the amount by which the final value of the mean square error, averaged over an ensemble of adaptive filters, deviates from the optimal minimum mean square error.
3. Computational complexity – This is the number of operations required to make one complete iteration of the algorithm.
4. Numerical properties – When an algorithm is implemented numerically, inaccuracies are produced due to round-off noise and representation errors in the computer. These kinds of errors influence the stability of the algorithm.

The radio channel characteristics and intended use of the subscriber equipment is another key issue. The speed of the mobile unit determines the channel fading rate and the Doppler spread, which is directly related to the coherence time of the channel. The choice of algorithm, and its corresponding rate of convergence, depends on the channel data and coherence time. We use LMS (least mean squares) and RLS (Recursive Least Squares) to produce a equalize object in Matlab name eqObj.

For adaptive algorithms, the equalizer may adapt in decision-directed mode using a detector specified by the property of equalizer or in training only mode using only pilot data to train the detector. In decision-directed mode, the equalizer uses a detected version of its output signal when adapting the weights. Adaptive equalizers typically start with training and switch to decision-directed mode after exhausting all symbols in the training sequence. The equalize function operates in decision-directed mode when one of these conditions is true: the syntax does not include a training sequence; or the equalizer has exhausted all symbols in the training sequence and still has more input symbols to process.

We may choose to reset the estimated weights of a equalizer or not before equalizer begin a new training cycle in next coher-



(a)



(b)

Figure 3: (a) original (b) Image quality of received
 ence time. If set ResetBeforeFiltering property to 1, during each
 coherence time, equalizer resets the state of equalizer come
 from training result of last coherence time. If 0, the equalization
 process uses the result of last coherence time either from
 training only mode or decision-directed mode.

III. SIMULATION AND EXPERIMENTAL RESULT

We discuss our simulation result by two steps. First we analyze the performance comparison by different parameter setting in each channel. Then we analyze the performance by comparing three different channels under the same parameters setting. All the simulations are based on QPSK modulation with gray code.

A. For AWGN channel

1) BER of simulation vs theoretical

As shown in figure 2, The BER performance of simulation result is closely identical to theoretical BER.

2) Image quality of received vs original

In figure 3, the received image is plot at SNR = 5dB, we see there are some random noises in the image. From simulation result, the received image quality is almost the same as original at SNR = 10dB.

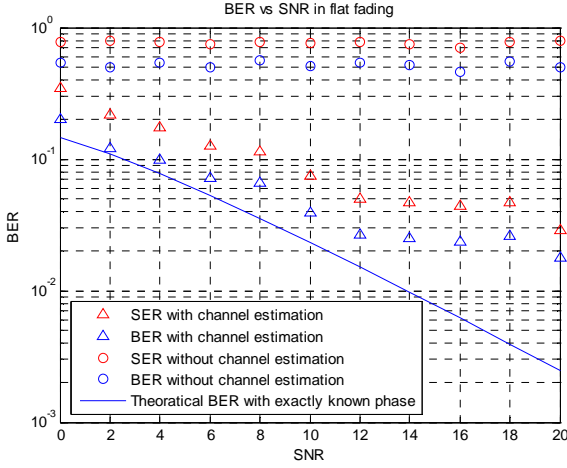


Figure 4: BER of simulation vs theoretical

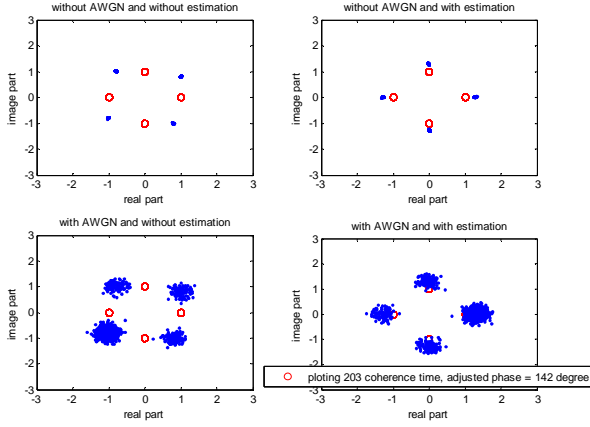


Figure 5: BER & constellation of training vs non-training

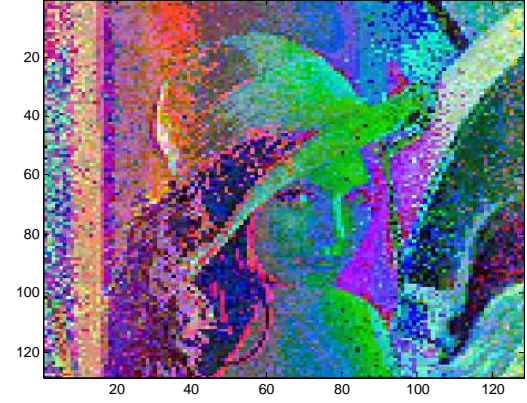
3) BER of Image vs random data

The correlation between image pixels does not effect the BER in AWGN channel.

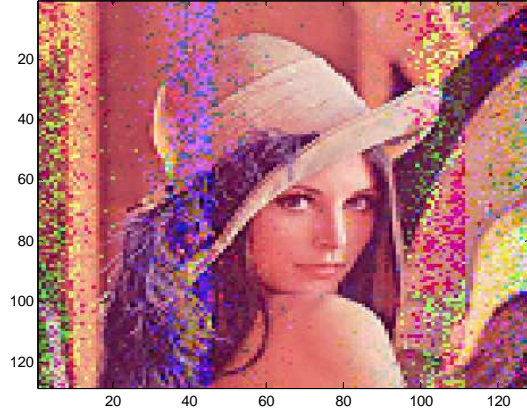
B. For flat fading channel

1) BER of simulation vs theoretical

As shown in figure 4, the BER performance of simulation result is worse than theoretical BER. This is reasonable, since the theoretical BER is based on the assumption that we know exactly the phase information of modulated signal. However, due to the time-variant channel, we always have estimation error for phase information. We also find the BER performance is improved dramatically in low SNR, while not in high SNR. This is also reasonable, since in low SNR, white Gaussian noise dominate the BER error, which can be improved by enhancing SNR, while in high SNR, phase estimation error dominate the BER error, which can not be improved by simply enhancing SNR.



(a)



(b)

Figure 6: (a) without adjustment (b) with adjustment

2) BER & constellation of training vs non-training

As shown in figure 4 and figure 5, the constellation is plot at SNR = 10dB, we see both the BER performance and constellation are greatly improved by channel phase estimation.

3) Image quality of received vs adjusted

In figure 6, the received image is plot at SNR = 10dB, we see that other than some random noise, there is some block noise in the image. This is due to the phase estimation error in a coherence time.

4) BER of Image vs random data

The correlation between image pixels does not affect the BER in flat fading channel.

C. For frequency selective fading channel

1) BER of simulation vs theoretical

As shown in figure 7, the BER performance of simulation result is worse than theoretical BER. The reason is same from above reason addressed in flat fading channel. Different from in flat fading channel, the BER performance is improved dramatically in low SNR, while even degraded in high SNR. This is

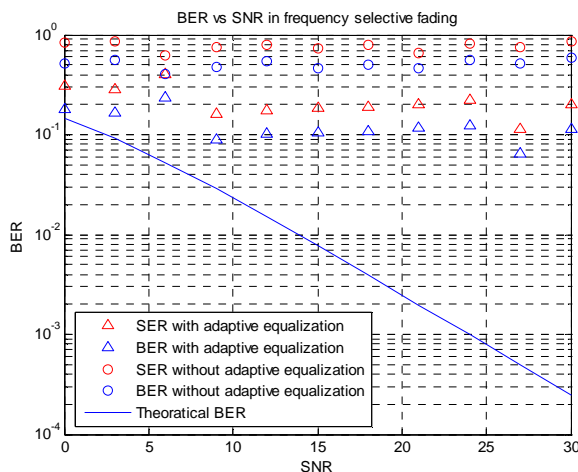


Figure 7: BER of simulation vs theoretical

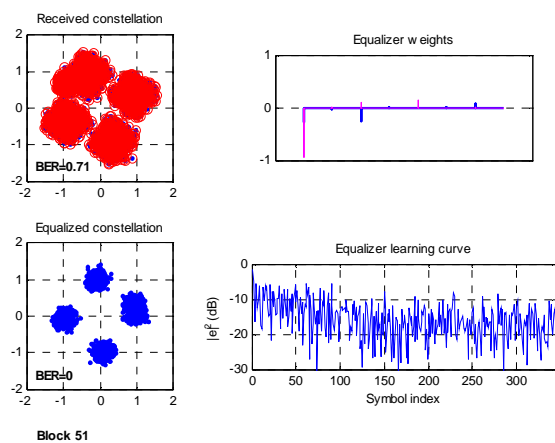
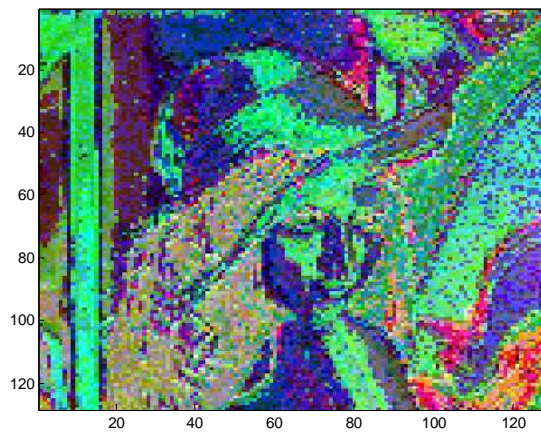


Figure 8: BER & constellation of equalized vs non-equalized



(a)



(b)

Figure 9: (a) without equalization (b) with equalization

also reasonable, since in high SNR, phase estimation error and ISI dominate the BER error, and the estimation error will cause even severe ISI., which cause the BER even worse.

2) BER & constellation equalized vs non-equalized

In figure 7 and figure 8, the constellation is plot at SNR = 15dB, we see both the BER performance and constellation are greatly improved by channel phase estimation.

3) Reset vs continue training result

The BER performances of resetting the state of equalizer come from training result of last coherence time is worse than using the result of last coherence time.

4) Training only vs decision directed mode

The BER is improved by using decision directed mode, since the time-variant property of the channel cause the channel change from estimation result of training data.

5) LMS vs RLS

The BER performances are almost same for both of them. But during the simulation, we find, LMS need more training data to

converge the equalizer comparing to RLS, while latter has more complexity and time consuming.

6) Image quality of received vs original

In figure 9, the received image is plot at SNR = 15dB, we see that other than some random noise and block noise in the image, there are some overlaps in the image. This is due to the white Gaussian noise, phase estimation error in a coherence time, and ISI caused by frequency selective fading channel.

7) BER of Image vs random data

The correlation between image pixels does not affect the BER in frequency selective fading channel, since we use PN code to train the equalizer.

D. Comparison among three channels

1) For Image comparison

In figure 11, we may see that in AWGN channel, the image is degraded by random noise; in flat fading channel, the image is degraded by random noise and block noise; in frequency selective fading channel, the image is degraded by random noise,

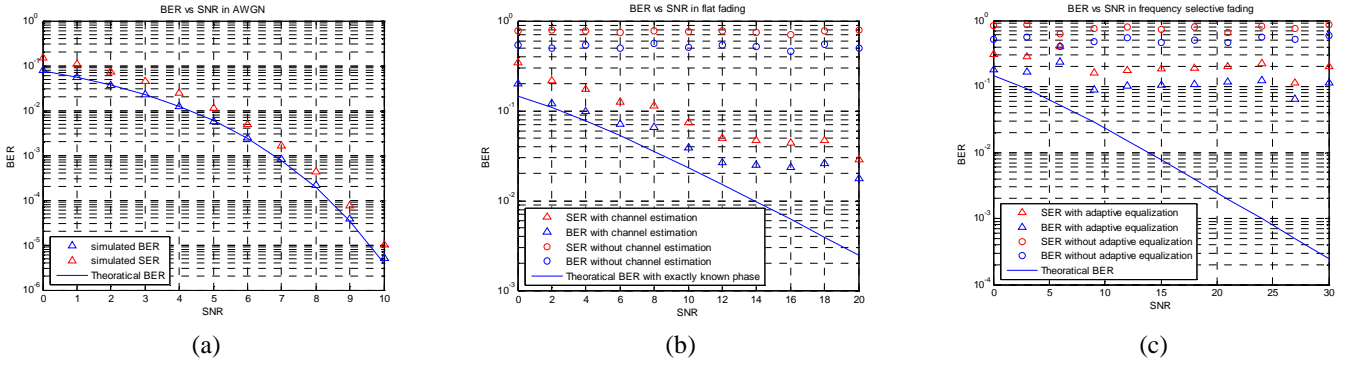


Figure 12: (a) AWGN channel (b) flat fading channel (c) frequency selective fading channel

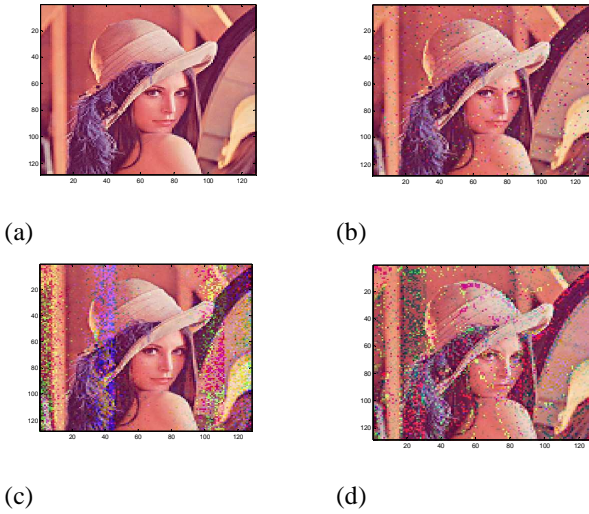


Figure 11: (a) original image (b) AWGN channel (c) flat fading channel (d) frequency selective fading channel

block noise, and overlap.

2) For BER performance comparison

As in figure 12, we see the BER performance is best in AWGN channel, worse in flat fading channel and worst in frequency selective fading channel. They are exactly as the theoretical analysis.

IV. FUTURE WORK

In this project, we choose PSK modulation to test the effect of different channels to the received data. So, we only estimate the channel phase information in channel estimation. We may add more modulation techniques in our model, such as ASK and QAM with different modulation orders. Then we will need to estimation both the channel phase information and amplitude information.

In flat fading channel, we train the detector by the pilot data in the head of source data in each coherence time. But the channel is time-variant even during one coherence time, so in our future simulation, we may use different interpolation algorithms between different coherence time to improve the estimated channel phase and amplitude information.

We use linear equalizer in our present model. As well know, linear equalizers do not perform well on channels which have

deep spectral nulls in the passband. While frequency selective fading channel normally causes the deep spectral nulls, so in our future simulation, we may improve this by add Decision Feedback Equalization (DFE).

In this project, we produce two different scenarios by simulate a GSM carrier frequency and bandwidth, and use pilot data to estimate the channel phase. All of these are simulated in Matlab at present. In our future model, we may integrate our model into GNU radio with USRP hardware support, which will give a practical environment to test our wireless communications simulation and our own algorithm.

V. CONCLUSION

In this paper, we test the effect of three different channel models, AWGN channel, flat fading channel, and frequency selective fading channel, to the data and image under two scenarios. We also compare and analysis the improvement of channel estimation and adaptive equalization in slow fading channel. Our result is exactly identical to the theoretical analysis. We also propose some possible improvements and probable future work, which will introduce more research interests.

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