Ripplet: a New Transform for Feature Extraction and Image Representation

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Outline

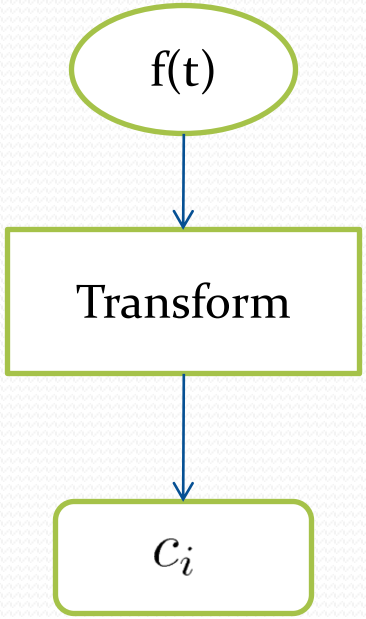
- Motivation
- Ripplet
  - Continuous ripplet transform
  - Discrete ripplet transform
- Experimental results
- Conclusions & future work

Transform representation of signal

- Function representation

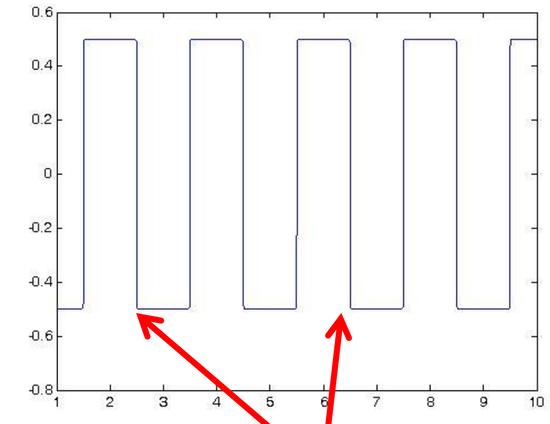
$$f(t) = \sum_i c_i \phi_i(t)$$

- Transforms with fixed bases
  - Fourier Transform
  - Wavelet Transform
  - Ridgelet Transform
  - …



Challenges in transform design

- Discontinuities (singularities) are difficult to be efficiently represented.
- Conventional solutions
  - Fourier transform -- Gibbs phenomenon.
  - Wavelet transform can resolve 1-D singularities, but it can not resolve 2-D singularities.

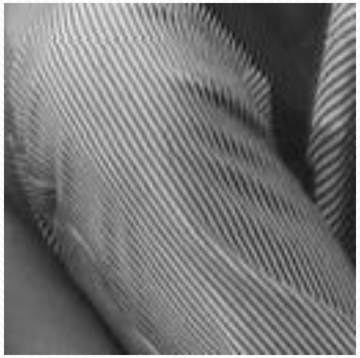


1D singularities

2D singularities

Existing solutions for resolving 2D singularities

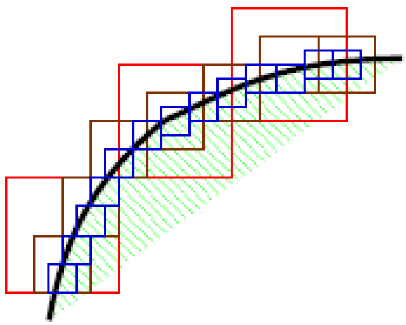
- Ridgelet [Candes and Donoho]
  - Resolve 2D singularities along lines
- Curvelet [Candes and Donoho]
  - Resolve 2D singularities along curves
- Contourlet [Do and Vetterli]
  - Resolve 2D singularities along curves



Properties of Curvelet

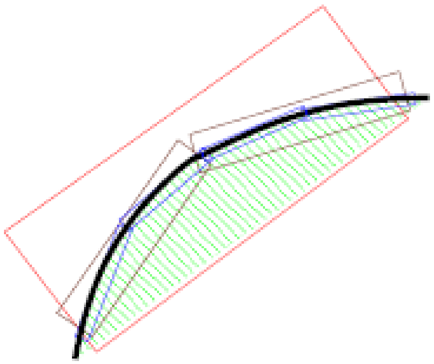
- Multi-resolution
- Directional
- Anisotropy:
  - Parabolic scaling provides anisotropy
  - Key difference from rotated 2-D wavelet.

Intuition



2-D wavelet
Square-shaped blocks

(Tensor product of
two 1-D wavelets)



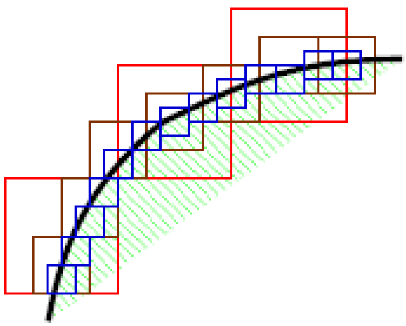
Contourlet
Rectangle-shaped blocks

Curvelet
Parabola-shaped blocks

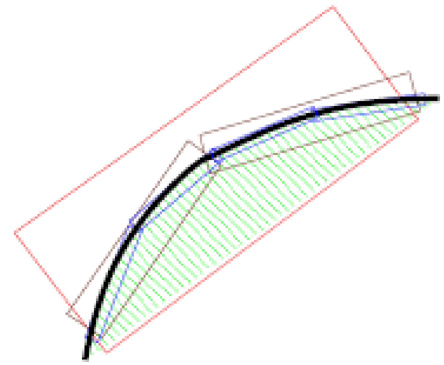
Conjecture

- Is the parabolic scaling law optimal for all types of boundaries?
- If not, what scaling law will be optimal?
- Our answer:
  - Generalize the scaling law ⇨ ripplet
  - Then, optimize over ripplets of different degrees and different support ranges

Intuition of Ripplet



Wavelet



Contourlet

Curvelet

Ripplet
Blocks with arbitrary shape and size

Ripplet Functions

- Ripplet functions:

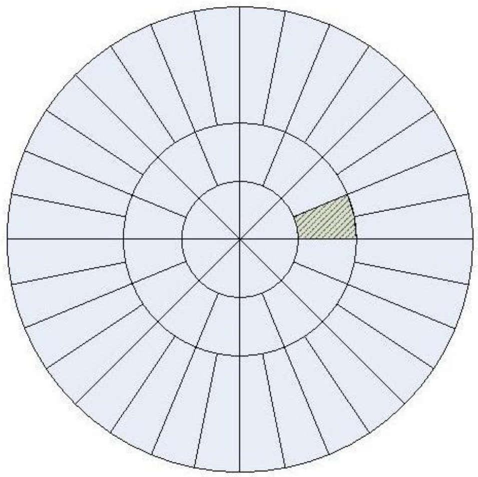
$$\rho_{a\vec{b}\theta}(\vec{x}) = \rho_{a\vec{0}0}(R_\theta(\vec{x} - \vec{b}))$$

  - Rotation matrix $R_\theta = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$
  - "Mother" function $\rho_{a\vec{0}0}(\cdot)$

Ripplet Functions (cnt'd)

- Ripplet mother function is defined in frequency domain

$$\hat{\rho}_a(r,\omega) = \frac{1}{\sqrt{c}} a^{\frac{d+1}{2d}} W(a \cdot r) V\left(\frac{a^{\frac{1}{d}}}{c \cdot a}\omega\right)$$



  - $\hat{\rho}_a(r,\omega)$ is the Fourier transform of $\rho_{a\vec{0}0}(\vec{x})$
  - $W(r)$ is "radial window" on $[1/2, 2]$
  - $V(\omega)$ is "angular window" on $[-1, 1]$
  - *c* determines the support
  - *d* denotes degree
  - Curverlet is just the special case of ripplet for $c = 1, d = 2$

Ripplet Functions in Space Domain

All ripplet functions are located in the center, i.e., $\vec{b} = 0$

$a = 3, \theta = 3\pi/16, c = 1, d = 2$

$a = 4, \theta = 3\pi/16, c = 1, d = 4$

$a = 3, \theta = 3\pi/16, c = 1.5, d = 2$

$a = 4, \theta = 3\pi/16, c = 1.5, d = 4$

Properties of Ripplets (1)

- Multi-resolution analysis
  - Ripplet transform provides a hierarchical representation of images. It can effectively approximate images from coarse granularity to fine granularity.
- High directionality
  - Ripplets can be pointed to arbitrary directions.

Properties of Ripplets (2)

- Good localization
  - Ripplets are well localized in both spatial and frequency domains.
- Arbitrary scaling
  - Ripplets allow scaling with arbitrary degree. The degree can take any real value. Curvelet is ripplet with degree 2.
- Anisotropy
  - Achieved by flexible scaling and arbitrary support range

Continuous Ripplet Transform

Forward transform:

$$R(a,\vec{b},\theta) = \int f(\vec{x})\overline{\rho_{a\vec{b}\theta}(\vec{x})}d\vec{x}$$

Backward transform:

$$\hat{f}(\vec{x}) = \int R(a,\vec{b},\theta)\rho_{a\vec{b}\theta}(\vec{x})dH$$

dH is the reference measure of $a,\vec{b},\theta$

Discrete Ripplet-I Transform

Substitute with discrete parameters

$$a_j = 2^{-j}$$

$$\vec{b_k} = [c \cdot 2^{-j} \cdot k_1, 2^{-j/d} \cdot k_2]^T$$

$$\theta_l = \frac{2\pi}{c} \cdot 2^{-\lfloor j(1-1/d) \rfloor} \cdot l \qquad j, k_1, k_2, l \in \mathbb{Z}$$

Forward transform:

$$R(j, \vec{k}, l) = \sum_{n_1=0}^{M-1} \sum_{n_2=0}^{N-1} f(n_1, n_2) \overline{\rho_{j,\vec{k},l}(n_1, n_2)}$$

Inverse transform:

$$\hat{f}(n_1, n_2) = \sum_{j} \sum_{\vec{k}} \sum_{l} R(j, \vec{k}, l) \rho_{j,\vec{k},l}(n_1, n_2)$$

Experimental Results

- Nonlinear approximation (NLA)
  - Sort coefficients in descending order

$$|c_0| \geq |c_1| \geq |c_2| \geq \cdots \geq |c_{n-1}| \geq |c_n| \geq \cdots$$

  - Approximate signal by n-largest coefficients

$$g \approx \hat{g} = \sum_{i=0}^{n-1} c_i \phi_i$$

  - Performance measure on reconstruction error

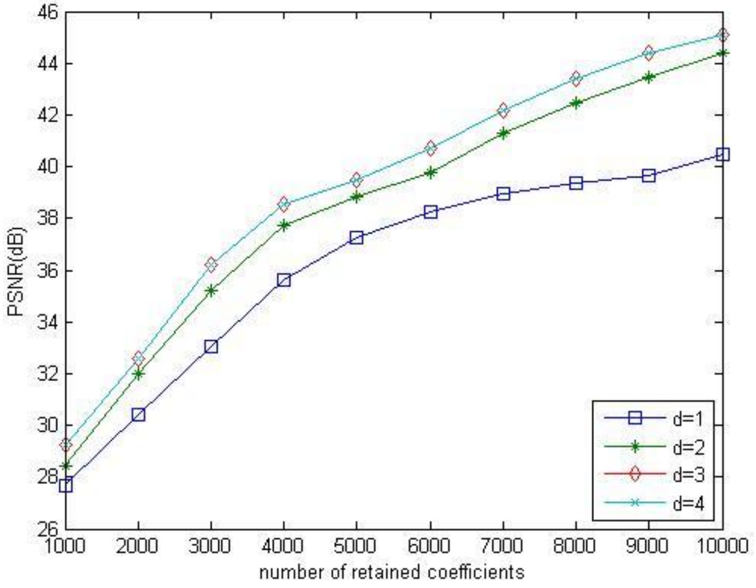
$$e = g - \hat{g}$$

  - Peak Signal Noise Ratio (PSNR)

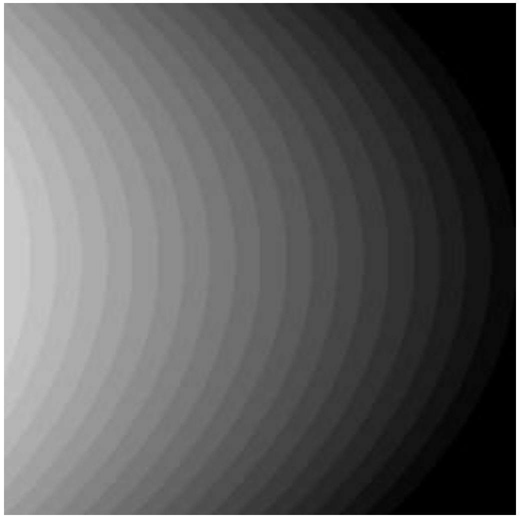
$$PSNR = 10 \times \log_{10}\left(\frac{1}{||e||_2^2}\right)$$

Synthetic Images (1)

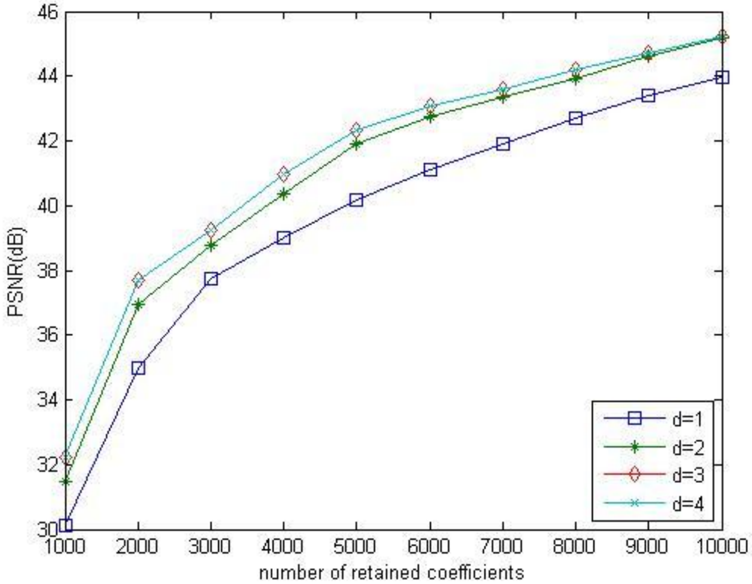
20 lines



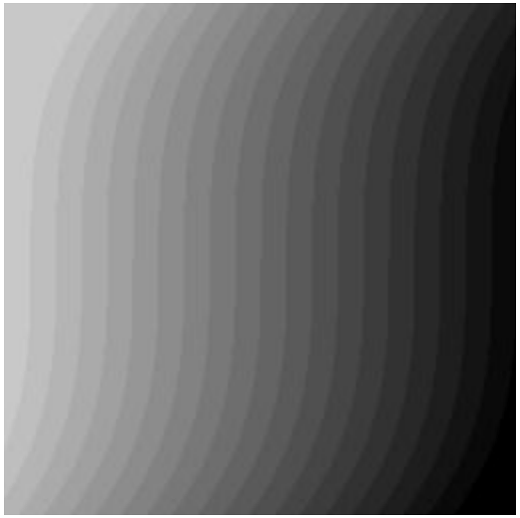
Synthetic Images (2)



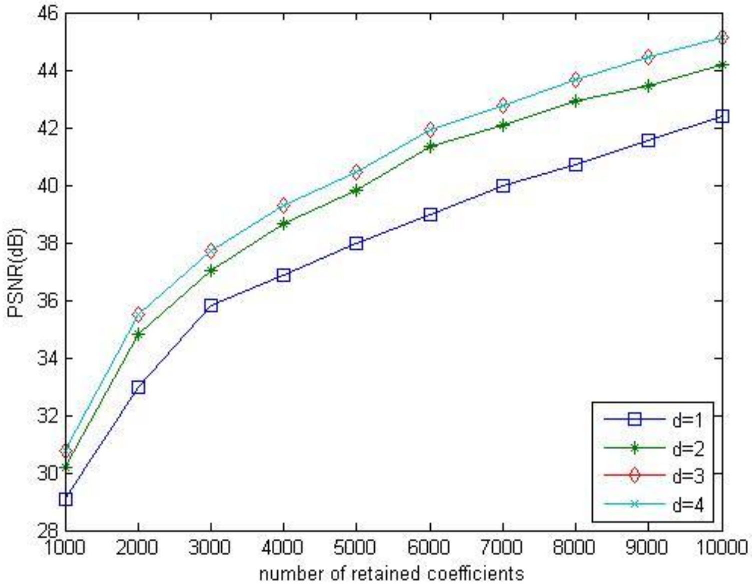
20 parabolic curves



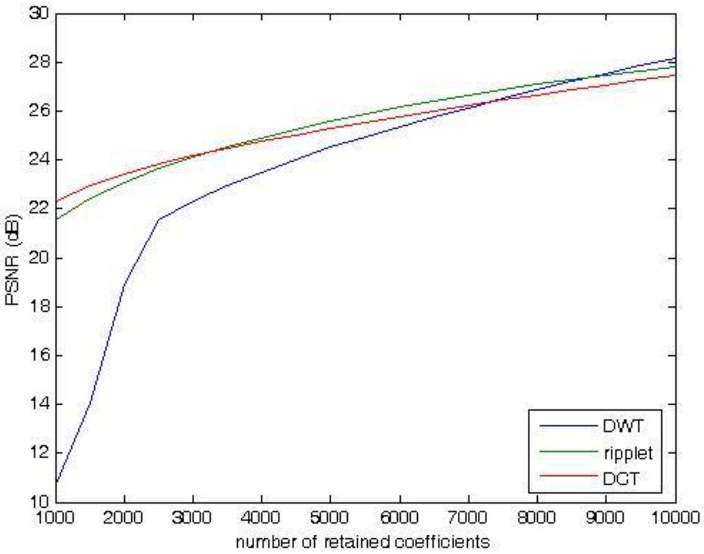
Synthetic Images (3)



20 cubic curves



Natural Images (1)

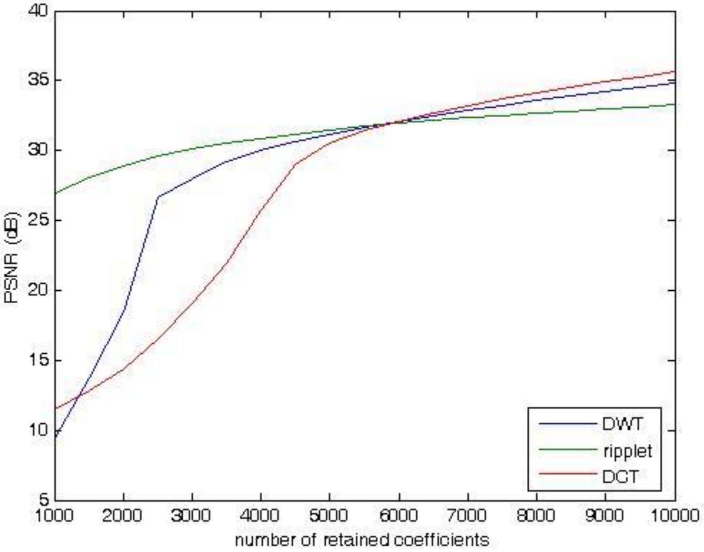


Natural Images (2)

Reconstructed with 5000 largest ripplet coefficients PSNR = 25.58 dB

Reconstructed with 5000 largest wavelet coefficients PSNR = 24.51 dB

Natural images (3)



Natural Images (4)

Reconstructed with 4000 largest ripplet coefficients PSNR = 31.13 dB

Reconstructed with 4000 largest wavelet coefficients PSNR = 30.13 dB

Conclusions & Future Work

- Ripplet transform can provide a more efficient representation of images with singularities along smooth curves.
- Ripplets have the capability of representing the shape of an object, but they are not good at representing textures.
- It is promising to combine ripplet and other transforms such as DCT to represent the entire image, which contains object boundaries and textures.

Thank you!