Ripplet: a New Transform for Feature Extraction and Image Representation

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Outline

- Motivation
- Ripplet
  - Continuous ripplet transform
  - Discrete ripplet transform
- Experimental results
- Conclusions & future work
Transform representation of signal

- Function representation
  \[ f(t) = \sum_i c_i \phi_i(t) \]

- Transforms with fixed bases
  - Fourier Transform
  - Wavelet Transform
  - Ridgelet Transform
  - ...

f(t) \rightarrow \text{Transform} \rightarrow C_i
Challenges in transform design

- Discontinuities (singularities) are difficult to be efficiently represented.
- Conventional solutions
  - Fourier transform -- Gibbs phenomenon.
  - Wavelet transform can resolve 1-D singularities, but it can not resolve 2-D singularities.
Existing solutions for resolving 2D singularities

- Ridgelet [Candes and Donoho]
  - Resolve 2D singularities along lines
- Curvelet  [Candes and Donoho]
  - Resolve 2D singularities along curves
- Contourlet [Do and Vetterli]
  - Resolve 2D singularities along curves
Properties of Curvelet

- Multi-resolution
- Directional
- Anisotropy:
  - Parabolic scaling provides anisotropy
  - Key difference from rotated 2-D wavelet.
Intuition

2-D wavelet
Square-shaped blocks
(Tensor product of two 1-D wavelets)

Contourlet
Rectangle-shaped blocks

Curvelet
Parabola-shaped blocks
Conjecture

- Is the parabolic scaling law optimal for all types of boundaries?
- If not, what scaling law will be optimal?
- Our answer:
  - Generalize the scaling law ripplet
  - Then, optimize over ripplet sets of different degrees and different support ranges
Intuition of Ripplet

Wavelet

Contourlet

Curvelet

Ripplet

Blocks with arbitrary shape and size
Ripplet Functions

- Ripplet functions:

\[ \rho_{a\vec{b}\theta}(\vec{x}) = \rho_{a\vec{0}0}(R_\theta(\vec{x} - \vec{b})) \]

- Rotation matrix

\[ R_\theta = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \]

- "Mother" function

\[ \rho_{a\vec{0}0}(\cdot) \]
Ripplet Functions (cnt’d)

• Ripplet mother function is defined in frequency domain

\[ \hat{\rho}_a(r, \omega) = \frac{1}{\sqrt{c}} a^{\frac{d+1}{2d}} W(a \cdot r) V\left(\frac{a^{\frac{1}{d}}}{c \cdot a}\right) \]

• \( \hat{\rho}_a(r, \omega) \) is the Fourier transform of \( \rho_{a00}(\vec{x}) \)
• \( W(r) \) is “radial window” on \([1/2, 2]\)
• \( V(\omega) \) is “angular window” on \([-1, 1]\)
• \( c \) determines the support
• \( d \) denotes degree
• Curverlet is just the special case of ripplet for \( c = 1, d = 2 \)
Ripplet Functions in Space Domain

All ripplet functions are located in the center, i.e., $\vec{b} = 0$

- $a = 3, \theta = 3\pi/16, c = 1, d = 2$
- $a = 4, \theta = 3\pi/16, c = 1, d = 4$
- $a = 3, \theta = 3\pi/16, c = 1.5, d = 2$
- $a = 4, \theta = 3\pi/16, c = 1.5, d = 4$
Properties of Ripplets (1)

- Multi-resolution analysis
  - Ripplet transform provides a hierarchical representation of images. It can effectively approximate images from coarse granularity to fine granularity.

- High directionality
  - Ripplets can be pointed to arbitrary directions.
Properties of Ripplets (2)

- Good localization
  - Ripplets are well localized in both spatial and frequency domains.

- Arbitrary scaling
  - Ripplets allow scaling with arbitrary degree. The degree can take any real value. Curvelet is ripplet with degree 2.

- Anisotropy
  - Achieved by flexible scaling and arbitrary support range
Continuous Ripplet Transform

Forward transform:

\[ R(a, \vec{b}, \theta) = \int f(\vec{x}) \rho_{a\vec{b}\theta}(\vec{x}) d\vec{x} \]

Backward transform:

\[ \hat{f}(\vec{x}) = \int R(a, \vec{b}, \theta) \rho_{a\vec{b}\theta}(\vec{x}) dH \]

\(dH\) is the reference measure of \(a, \vec{b}, \theta\)
Discrete Ripplet-I Transform

Substitute with discrete parameters

\[ a_j = 2^{-j} \]

\[ \vec{b}_k = \begin{bmatrix} c \cdot 2^{-j} \cdot k_1, 2^{-j/d} \cdot k_2 \end{bmatrix}^T \]

\[ \theta_l = \frac{2\pi}{c} \cdot 2^{-\lfloor j(1-1/d) \rfloor} \cdot l \quad j, k_1, k_2, l \in \mathbb{Z} \]

Forward transform:

\[ R(j, \vec{k}, l) = \sum_{n_1=0}^{M-1} \sum_{n_2=0}^{N-1} f(n_1, n_2) \rho_{j, \vec{k}, l}(n_1, n_2) \]

Inverse transform:

\[ \hat{f}(n_1, n_2) = \sum_j \sum_{\vec{k}} \sum_l R(j, \vec{k}, l) \rho_{j, \vec{k}, l}(n_1, n_2) \]
Experimental Results

- **Nonlinear approximation (NLA)**
  - Sort coefficients in descending order
    \[ |c_0| \geq |c_1| \geq |c_2| \geq \cdots \geq |c_{n-1}| \geq |c_n| \geq \cdots \]
  - Approximate signal by $n$-largest coefficients
    \[
    g \approx \hat{g} = \sum_{i=0}^{n-1} c_i \phi_i
    \]
  - Performance measure on reconstruction error
    \[
    e = g - \hat{g}
    \]

- **Peak Signal Noise Ratio (PSNR)**
  \[
  PSNR = 10 \times \log_{10}\left(\frac{1}{\|e\|_2^2}\right)
  \]
Synthetic Images (2)

20 parabolic curves
Synthetic Images (3)

20 cubic curves
Natural Images (1)
Natural Images (2)

Reconstructed with 5000 largest ripplet coefficients PSNR = 25.58 dB

Reconstructed with 5000 largest wavelet coefficients PSNR = 24.51 dB
Natural images (3)
Natural Images (4)

Reconstructed with 4000 largest ripplet coefficients PSNR = 31.13 dB

Reconstructed with 4000 largest wavelet coefficients PSNR = 30.13 dB
Conclusions & Future Work

- Ripplet transform can provide a more efficient representation of images with singularities along smooth curves.
- Ripplets have the capability of representing the shape of an object, but they are not good at representing textures.
- It is promising to combine ripplet and other transforms such as DCT to represent the entire image, which contains object boundaries and textures.
Thank you!