**Differential Topology**

* Conformal map between two topological spaces:

Let us say we have a two Riemannian manifolds *M* and *M*′ of dimension 2 with a diffeomorphism *f*:*M*→*M*′ between them. Say *f* is conformal, i.e., for every point *p*∈*M*, there is a positive number *a*(*p*) such that

⟨*f*∗(*u*),*f*∗(*v*)⟩*M*′,*f*(*p*)=*a*(*p*)⟨*u*,*v*⟩*M*,*p*

for all *u*,*v*∈*TpM*. We must determine the relationship between the Gaussian curvatures between the two manifolds.

* + Intuition: From the definition, we know that “the ratio of two metrics (or the ratio of lengths of two tangent vectors) is constant”: the ratio ⟨*f*∗(*u*),*f*∗(*v*)⟩*M*′,*f*(*p*) / ⟨*u*,*v*⟩*M*,*p* is constant a(p) for all *u*,*v*∈*TpM*. This implies that the ratio of the lengths of any two corresponding edges on a triangle is constant => the two corresponding triangles in two tangent spaces *TpM* and *Tf(p)M*′ are similar => the corresponding angles in two tangent spaces are the same. That is, f preserves angle, i.e., f is conformal.
  + Proof: cos(*f*∗(*u*),*f*∗(*v*)) = ⟨*f*∗(*u*),*f*∗(*v*)⟩/sqrt{⟨*f*∗(*u*),*f*∗(*u*)⟩ \* ⟨*f*∗(*v*),*f*∗(*v*)⟩} = a\* ⟨*u*,*v*⟩/sqrt{a\*⟨*u*,*u*⟩ \* a\*⟨*v*,*v*⟩} = ⟨*u*,*v*⟩/sqrt{⟨*u*,*u*⟩ \* ⟨*v*,*v*⟩}=cos(u,v)
  + Different point *p*∈*M* can have different *a*(*p*). Hence, by changing each *a*(*p*) appropriately, we can obtain a Ricci flow, which eventually leads to M′ that has a constant curvature at every point *q*∈*M*′ .
* Vertex scaling in discrete uniformization theorem proof, discrete conformal equivalence
* Theorem Egregium

Theorem Egregium in terms of forms, where for an orthonormal frame *e*1,*e*2 we have the Gaussian curvature is given by

*K*=Ω12(*e*1,*e*2)

where Ω12 is a curvature 2-form. We also have that the Gaussian curvature at a point is given by

*Kp*=⟨*Rp*(*u*,*v*)*v*,*u*⟩

for any orthonormal basis *u*,*v* for *TpM*. Further, we know that

⟨*R*(*e*1,*e*2)*e*2,*e*1⟩= Ω12 (*e*1,*e*2).