Section 1-Wave Fundamentals

1.1 What is a wave?

• Encounter waves in many situations
  - Speech and hearing rely on wave propagation.
  - Modern telecommunication networks such as mobile phones rely on waves.
  - Many key areas of Physics, Mathematics and Chemistry are best described by waves and their interactions.
  - The way atoms bind together to form molecules can be understood by the overlap of waves.
What can waves do?

- A wave can do many things.
- Travel e.g. pulse on a string, telecommunications signal down optical fibre.
- Carry energy and momentum from one point to another.
- Bounce off surfaces - reflection.
- Go across boundaries - refraction.
- Go round corners - diffraction.
- Interact and superimpose - interference.
- Change shape - dispersion.
- Loose energy - dissipation.
The definition of a wave

• As a wave can do many different things we must come up with a suitable definition for a wave.

• A wave is a disturbance from a normal or equilibrium condition that propagates without the transport of matter.

• e.g. Mexican wave - wave appears to move round stadium as people move up and down in their seats.
  - Equilibrium condition - people sitting down.
  - Disturbance - people standing up.
Types of waves

- There are several different types of wave that we must consider.

  - **Mechanical Waves**:- These need a medium to propagate in - *sound waves*.

  - **Non-mechanical waves**:- These waves do not need a medium in which to propagate - *light waves*.

  - **Matter waves**:- Particles such as protons and electrons can be treated as waves. This forms the basis of quantum mechanics. We will not be discussing this type of wave in this course.
Classification of waves

- A wave can be classified by the way it disturbs the medium through which it propagates.
- **Transverse waves**: The displacement is at right angles to the direction of propagation — light waves.
Transverse waves

As the pulse approaches the point marked 0 the particle at 0 is at rest.

Once the pulse reaches the particle, the particle is displaced from its rest point.

When the peak of the pulse reaches the particle, the particle experiences its maximum displacement.

Direction of propagation
Transverse waves

After the peak of the pulse has passed through the point 0 the particle starts to return to its rest position. This happens because the displacement induced by the pulse decreases.

Once the pulse has completely propagated through the point 0 the particle at 0 remains at rest.
Classification of waves

**Longitudinal waves**: The displacement is in the same direction as the wave propagates - *sound waves*.
1.2 Mathematical Description of a wave

Need to make several assumptions before we can describe a wave.

1. Wave depends on both position, \( x \), and time, \( t \).
2. We have a random disturbance \( y(x,t) = f(x,t) \).
3. Wave travels in straight line in \( x \) direction.
4. Wave travels at a constant speed \( v \).
5. Wave does not change shape - Non-dispersive.
6. Wave does not lose energy - Non-dissipative.
7. Need to define a frame of reference to understand pulse propagation.
1.2 Mathematical Description of a wave

• A frame of reference allows us to quantify processes. The speed of a car is measured with respect to the road. The road is the frame of reference.

• For a pulse there are two possible frames of reference

1) Laboratory Frame of Reference:-

   Here we define at set of axes $x,y,z$ and as time changes the pulse moves away from the origin.

2) Pulse Frame of Reference:-

   Here we define a set of axes $x',y',z'$ that move with the pulse and at the same speed as the pulse. The position of pulse is stationary in this frame and so pulse is time invariant, i.e. independent of time.
At time \( t = 0 \) s both frames of reference coincide. Consider two points, \( x \) in laboratory frame and \( x' \) in pulse frame.

After time \( t \), pulse in laboratory frame moves and point is now \( \Delta x \) from origin.

In pulse frame the point \( x' \) is unchanged but axes have moved a distance \( vt \).
1.2 Mathematical description of a wave

Is there anyway we can relate the two frames of reference?

Let us look at pulse in the pulse frame of reference

• Pulse is described by an arbitrary function.

\[ y'(x') = f(x') \]

• The pulse has the same profile irrespective of the frame of reference. So in the laboratory frame of reference

\[ y(x,t) = f(x') \]

• It is easy to show that

\[ \partial x = x' + vt \]

\[ x' = \partial x - vt \]
In the **Laboratory Frame**, the position is denoted as \( Y(x,t=0s) \) and \( Y(Dx,t=t) \), where \( D \) represents the displacement.

In the **Pulse Frame**, the position is denoted as \( Y'(x') \) and \( Y'(x') \), with \( x' \) representing the position in the pulse frame.

The diagram illustrates the transformation of a pulse from the laboratory frame to the pulse frame, showing the change in position over time.
1.2 Mathematical description of a wave

So

\[ y(x,t) = f(D_x - vt) \]

But \( D_x \) is an arbitrary variable that can be replaced with \( x \).

\[ y(x,t) = f(x - vt) \]

This equation means that in the laboratory frame of reference, we are always looking at the same point on the pulse.

Thus the general equation for a wave that moves at a speed \( v \) is

\[ y(x,t) = f(x - vt) \]

This is known as the wave function.
1.3 Key features of the wave equation

The general equation for a wave that moves at a speed $v$ is

$$y(x,t) = f(x - vt)$$

• The quantity $x - vt$ is known as the **phase** of the wave.
• The phase gives us information about the displacement from the $x$-axis.
• The speed $v$ at which the disturbance propagates is called the **wave speed** or **wave velocity**.
• The $-$ sign tells us that the wave moves from **left to right**.
• A $+$ sign tells us that the wave moves from **right to left**.

$$y(x,t) = f(x - vt) \quad \text{Left to right}$$

$$y(x,t) = f(x + vt) \quad \text{Right to left}$$
When considering the wave function
\[ y(x, t) = f(x - vt) \]
For this to represent a traveling wave the three quantities \( x, v \) and \( t \) must appear in linear combinations. If this was not the case then as the pulse moved in time and space we would not observe the same point on the pulse.

Hence \((x - vt)^3\) is valid but \((x^3 - (vt)^3)\) is not valid.
Which of the following wave functions corresponds to traveling waves?

- (a) $y(x,t) = \sin^2(\pi(t-x/v))$
- (b) $y(x,t) = \cos((x-vt)^2)$
- (c) $y(x,t) = \sin((x)^2-(vt)^2)$
- (d) $y(x,t) = \exp(-s(x-vt)^2)$
- (e) $y(x,t) = (x+vt)^3$
- (f) $y(x,t) = \exp(-at) \cos((x-vt))$

Ans (a),(b),(d) and (e)